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# Absolutely Uniform Illumination of Laser Fusion Pellets

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Absolutely uniform illumination of spherical laser fusion pellets is possible when the energy deposition from a single laser beam is given by a simple $\cos^2\theta$ distribution. Conditions can be derived for which the laser beam targeting angles allow this absolute illumination uniformity. Configurations based upon the cube and higher order Platonic solids satisfy the constraints, as well as an infinite class of other less symmetric configurations.

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# ABSOLUTELY UNIFORM ILLUMINATION OF LASER FUSION PELLETS

For direct driven laser fusion to be successful, the spherical fuel pellet must be illuminated to a high degree of uniformity.[1]. Various symmetric beam-target configurations have been proposed to achieve this uniformity. Most of these designs are based on laser beams that are targeted on the spherical pellet from the faces or vertices of the five Platonic solids (the tetrahedron, cube, octahedron, dodecahedron, and icosahedron).[2-5] Given such symmetric targeting of the laser beams, the level of uniformity still depends on the characteristics of the laser beams (spot size on target, intensity profile) and pellet (plasma scalelength, absorption, and refraction). A brief derivation is introduced here that proves that absolutely uniform illumination from multiple beam configurations is possible in certain cases. These cases include configurations based on four of the five Platonic solids (the tetrahedron is excepted), plus an additional class of configurations that are less symmetric.

Assume, for the moment, that refractive effects are ignorable. This is best satisfied when the plasma scalelength is small in comparison to the pellet radius and the plasma is highly absorbing, a situation that accurately describes shorter wavelength experiments. The absorbed energy density on the pellet surface resulting from a single laser beam incident along the positive z axis is:

$$E(\theta,\phi) = I[x(\theta,\phi),y(\theta,\phi)]A(\theta)\cos(\theta) \qquad (1)$$

where $A(\theta)$ is the fraction of incident energy absorbed at polar angle $\theta$. The beam is implicitly assumed to result from large F number optics, so that the

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beam intensity profile, I, remains unchanged along the axis z in the area of the pellet. The total energy density is the superposition of many beams centered at the points $(\theta,\phi)$ on the pellet surface, and can be written as

$$E(\theta,\phi) = \sum_i I_i\ (\underline{x}_{i\perp})A(\gamma_i)\cos\gamma_i \tag{2}$$

$\gamma_i$ is the angle between the i-th beam axis and the point $(\theta,\phi)$, and $I(\underline{x}_{i\perp})$ is the beam intensity profile of the i-th beam in the plane orthogonal to its propagation axis.

Assume that all of the beams are identical, symmetric about their axis of propagation (so that $I_i(\underline{x}_{i\perp}) = I(\gamma_i)$), and that each beam is opposed by another that is antiparallel to it and aimed at the point directly opposite on the sphere, $(\pi-\theta_i,\pi+\phi_i)$. Then for the special case that the beam intensity profile and the pellet absorption function combine so that:

$$I(\gamma_i)A(\gamma_i) = I_o\cos\gamma_i \tag{3}$$

all angular dependence can be removed from the energy density on the pellet surface.

To show this, insert (3) in (2) and use the identity $\cos\gamma_i = \cos\theta\cos\theta_i + \sin\theta\sin\theta_i\cos(\phi_i-\phi)$. Rearranging the results, one finds:

$$E(\theta,\phi)/I_o = (\sum_k J_k\ \cos^2\theta_k)\cos^2\theta$$

$$+ (\sum_k\sin^2\theta_k\ \sum_j\sin^2\phi_{jk})\sin^2\theta\sin^2\phi$$

$$+ (\sum_k\sin^2\theta_k\ \sum_j\cos^2\phi_{jk})\sin^2\theta\cos^2\phi$$

$$+ 2(\sum_k\cos\theta_k\sin\theta_k\ \sum_j\cos\phi_{jk})\cos\theta\sin\theta\cos\phi$$

$$+ 2(\sum_k \cos\theta_k \sin\theta_k \sum_j \sin\phi_{jk})\cos\theta\sin\theta\sin\phi$$

$$+ 2(\sum_k \sin^2\theta_k \sum_j \cos\phi_{jk}\sin\phi_{jk})\sin^2\theta\cos\phi\sin\phi \tag{4}$$

The targeting angles $(\theta_k, \phi_{jk})$ have been written so that a group of azimuthal angles $\{\phi_{jk}; j=1,\ldots,J_k\}$ is associated with each polar angle $\theta_k, k=1,\ldots,K$. The total number of beams is thus $N = 2\sum_k J_k$.

The first requirement in achieving energy uniformity is that the last three terms of (4) are zero. This will occur when

$$\sum_j \sin\phi_{jk} = \sum_j \cos\phi_{jk} = \sum_j \cos\phi_{jk} \sin\phi_{jk} = 0 \tag{5}$$

for each k. This is satisfied if the set of angles, $\{\phi_{jk}\}$, are symmetric in reflections about the x and y axes. It is also satisfied when $\{\phi_{jk}\}$ are equally distributed about $2\pi$:

$$\{\phi_{jk}\} = 2\pi j/J_k \qquad j=1,\ldots,J_k \tag{6}$$

This includes the case of odd $J_k$ (see appendix A).

Next, the angles $\{\phi_{jk}\}$ are further constrained by the condition:

$$\sum_j \cos^2\phi_{jk} = \sum_j \sin^2\phi_{jk} = J_k/2 \tag{7}$$

The $\{\phi_{jk}\}$ given by (6) satisfy this condition if $J_k > 2$ (see appendix B). With this requirement satisfied, the energy density is independent of azimuthal angle and is given by:

$$E(\theta,\phi)/I_o = (\sum_k J_k \cos^2\theta_k)\cos^2\theta + 1/2(\sum_k J_k \sin^2\theta_k)\sin^2\theta \tag{8}$$

When the coefficients of the $\cos^2\theta$ and $\sin^2\theta$ terms are equal, the energy density becomes independent of polar angle also. This requirement reduces to:

$$\sum_k J_k(\cos^2\theta_k - 1/3) = 0 \tag{9}$$

If conditions (9), (7), and (5) hold, then

$$E(\theta,\phi)/I_o = N/6 \tag{10}$$

and the energy density on the pellet has no angular dependence.

The simplest configuration for which the conditions (5), (7), and (9) can be satisfied is K=1 and J=3: $\{\phi\}$ = 0, 120, 240°, and $\theta$ = 54.736°. These angles are in the direction of the faces of a cube, which is the simplest of Platonic solids with opposing beam symmetry. The other higher order Platonic solids also satisfy the uniformity constraints, as well as an infinite number of other less symmetrical configurations. For example, the University of Rochester's 24 beam Omega system[3] (aiming from the vertices of a small regular rhombicuboctahedron) belongs to this uniformity class. Lawrence Livermore Laboratory's NOVA laser system,[5] consisting of 5 sets of opposing beams ($\{\phi\}$ = 0, 72, 144, 216, 288°, and $\theta$ = 50°) is almost a member of this class (it would belong if $\theta$ = 54.736°). In general, given K > 1 and $J_k$, there exists an infinite set of configurations that satisfy the requirements.

It is possible to extend these results to more realistic situations. Refraction considerations give rise to more general and complex conditions than given by (1) through (3); the energy density resulting from a single laser beam must satisfy:

$$E(\theta,\phi) = A(\theta)\int d\theta' I(\theta')f(\theta',\theta)\cos\theta' = I_o\cos^2\theta \tag{11}$$

where $f(\theta',\theta)$ is the transmitted fraction of a ray originally incident at angle $\theta'$ and refracted to the angle $\theta$. As a practical example, condition (11) can be satisfied by a totally uniform beam incident upon an inverse square electron density distribution.[4]

In general, there is little control possible over the absorption or refraction of the light once it arrives on the pellet. Instead, some tailoring of the absorbed energy may possibly be accomplished through control of the laser intensity profile, perhaps by using apodized aperatures or other spatial filtering methods.

The question of inter-beam interference effects should also be considered. For coherent laser beams separated by a minimum angle $\psi$, the spatial intensity fluctuations created by interference patterns have a maximum spatial wavelength, $\lambda$, given by $\lambda = \lambda_o/\sin\psi \sim \lambda_o$ where $\lambda_o$ is the laser wavelength, and $\sin\psi$ is of order unity for typical systems.[6] Any nonuniformities in the deposited energy produced by this highly modulated intensity distribution are easily smoothed by thermal conduction in the region separating the absorption and ablation regions of the laser fusion pellet.[1] Interference effects are unimportant for incoherent illumination sources, including induced incoherent laser beams[7] as well as ion or electron beams.

In conclusion, it has been shown here that it is theoretically possible to illuminate a sphere in an absolutely uniform manner. The conditions for doing so are: 1) that the energy density resulting from any one laser beam is given by (11); 2) that to each beam belongs an opposing beam situated at the opposite point on the sphere; and 3) that the beam targeting angles satisfy the conditions (5), (7), and (9). These three constraints are satisfied for an

infinite set of configurations, including the cube and higher order Platonic solids. These configurations can be used to achieve the high degree of energy deposition uniformity required for proper implosion of laser fusion fuel pellets.

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2. J.E. Howard, Appl. Opt. 16, 2764 (1977).

3. S. Skupsky, K. Lee, Univ. of Rochester Report LLE-137 (1983).

4. I.N. Ross, J. Phys. D: Appl. Phys. 16, 1245 (1983).

5. J.B. Trenholme and E.J. Goodwin, Lawrence Livermore Laboratory Laser Program Annual Report UCRL-50021-79, p. 2-139 (1979).

6. For $N \gg 1$, $\psi$ approaches $4/\sqrt{N}$ for approximately symmetric placement of the beams, thus $\sin\psi > 1/10$ for $N < 1000$.

7. R.H. Lehmberg and S.P. Obenschain, Opt. Comm. 46, 27 (1983).

## Appendix A

If $\{\phi_j\}$ is symmetric about reflections in the x and y axis, then for each $\phi_j$, there exists a $\phi'_j$ such that $\phi'_j = -\phi'_j$, and another such that $\phi'_j = \pi - \phi_j$. Then clearly $\sum_j \sin\phi_j = 0$, $\sum_j \cos\phi_j = 0$, and $\sum_j \cos\phi_j \sin\phi_j = 0$. The $\{\phi_j\}$ given by (6) for even J is obviously a special case of this. A separate situation exists when J is odd. In this case, $\sum_j \sin\phi_j = 0$ and $\Sigma_j \sin\phi_j \cos\phi_j = 0$, since the set is symmetric in reflection about the x axis. In addition,

$$\sum_j \cos(2\pi j/J) = 1/2\left\{\sum_j [\cos(2\pi j/J) + \cos(2\pi [j-1]/J)]\right\} \tag{A-1}$$

$$= \cos(\pi/J)\sum_j \cos(2\pi [j-1/2]J)$$

The set of angles in the sum on the right side is the reflection of the original set about the y axis. But these sets are antisymmetric in reflection about the y axis:

$$\sum_j \cos(2\pi j/J) = -\sum_j \cos(2\pi j/J + \pi) = -\sum_j \cos(2\pi [j-1/2]/J) \tag{A-2}$$

Thus, if $\cos(\pi/J) \neq 1$ (or $J \neq 1$), we conclude that $\sum_j \cos(2\pi j/J) = 0$, and therefore (6) satisfies condition (5) for odd and even J.

## Appendix B

To show that (6) satisfies (7), note that

$$\sum_j \cos^2(2\pi j/J) = 1/2(J + \sum_j \cos(4\pi j/J)) \tag{B-1}$$

If J is even ($J=2\Gamma$, where $\Gamma$ is integer), then

$$\sum_j \cos^2(2\pi j/J) = 1/2(J + \sum_j \cos(2\pi j/\Gamma)) - J/2 \tag{B-2}$$

according to the results in Appendix A.

If J is odd, then

$$\sum_j \cos^2(2\pi j/J) + \sum_j \cos^2(\pi - 2\pi j/J) = J \tag{B-3}$$

by (B-2). But

$$\sum_j \cos^2(2\pi j/J) = \sum_j \cos^2(\pi - 2\pi j/J) \tag{B-4}$$

and therefore

$$\sum_j \cos^2(2\pi j/J) = J/2 \tag{B-5}$$

Thus, $\{\phi_j\}$ given by (6) satisfied condition (7) for even and odd J.