Nonlinear modelling of start-up phase pressure spectra from optically smoothed induced spatial incoherence laser imprint

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The spectrum of early time pressure perturbations, due to optically smoothed induced spatial incoherence (ISI) laser imprint, is computed for a planar target using a forced, dissipative model. The time-dependent ISI laser deposition is computed using a time-dependent electromagnetic full wave Maxwell code. It is found that the pressure spectrum evolves into a power law in which spectral power is transferred from large to smaller scales through a nonlinear cascade process. The model results are compared with experimental observations.

I. INTRODUCTION

A limitation on high-gain direct drive inertial confinement fusion is pellet degradation and disruption due to the Rayleigh-Taylor and Richtmyer-Meshkov instability[1]. An important issue is the nature of the seeding of these instabilities. This seeding can be generated by pellet surface imperfections and laser nonuniformities, i.e., laser imprint.

Much work, both experimental[2–9] and theoretical[10–18], has been performed dealing with laser imprint. We define the imprint phase as the period before the return of the first rarefaction wave to the ablation surface after shock breakout. It has been demonstrated that the size of the conduction zone between the critical surface and the ablation surface is important for smoothing laser nonuniformities. In the start up phase at early times before there is a large separation between the critical and ablation surface several studies have shown that the larger scale modes seem to dominate the smaller scale modes[6,8]. To minimize the effects of imprint, much work has been directed at improving the spatial uniformity of the laser irradiance, e.g., using optical smoothing methods such as ISI[19]. Some nonuniformity remains even after smoothing takes place.

In the time-dependent start-up phase the pressure profile will approximately match the driving laser intensity profile. For the seeding problem, it is important to develop a model for the spectrum of pressure perturbations over a wide range of spatial scales at early times. Nonlinear effects may be important in the establishment of the early time pressure spectrum. In this study we develop a nonlinear model of the pressure perturbation spectrum due to optically smoothed induced spatial incoherence (ISI) laser imprint. We find that in the pressure spectrum a nonlinear cascade develops where energy is transferred from the large scales down to small scales.

II. MODEL

We consider a planar fully ionized plasma with the initial density and temperature profiles shown in Fig. 1. The laser is incident from the left. Using a hydrodynamic fluid model the interaction of a high intensity laser with a planar target can be described using a complete set of fluid equations which are coupled to Maxwell's equations. It is not possible to solve this coupled model on both laser waveperiod femtosecond and hydrodynamic nanosecond time scales simultaneously. Instead, we adopt a forced, dissipative model to simulate the hydrodynamic evolution. The forced, dissipative approach has been widely used in hydrodynamic turbulence studies. This approach allows the study of aspects of the fundamental question of coupling among a range of spatial scales and the spectrum of pressure perturbations. We adopt the following equations for density, momentum, and energy:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} \rho V_j = 0 \qquad (1)$$

$$\frac{\partial}{\partial t}\rho V_j + \frac{\partial}{\partial x_l}\left(\rho V_j V_l\right) = \frac{\partial p}{\partial x_j} + \eta\frac{\partial}{\partial x_l}\left(W_{jl} - \frac{1}{3}\Delta\delta_{jl}\right) \quad (2)$$

$$\frac{\partial}{\partial t}\frac{p}{\gamma-1} + p\frac{\partial V_j}{\partial x_j} + \frac{\partial}{\partial x_j}\frac{pV_j}{\gamma-1} = -\nabla\cdot\mathbf{q} + \mathbf{J}\cdot\mathbf{E} \quad (3)$$

where $p = \rho T = p_e + p_i$, $W_{jl} = 1/2(\partial V_l/\partial x_j + \partial V_j/\partial x_l)$, $\Delta = \nabla\cdot\mathbf{V}$, $\mathbf{q} = -\kappa\nabla T$, η is the viscosity, κ is the Spitzer thermal conductivity, δ_{ij} is the Kronecker delta, and γ is the specific heat ratio. Here $\mathbf{J}\cdot\mathbf{E}$ gives the laser energy deposition due to collisional and resonance absorption. Using a collisional fluid model, $\mathbf{J}\cdot\mathbf{E} \simeq \sigma_r\mathbf{E}^2 \simeq \sigma_r I$ with $\sigma = i\omega_{pe}^2/(4\pi(\omega + i\nu_{ei}))$ where ω_{pe} is the plasma frequency, ω is the laser frequency, ν_{ei} is the collision frequency, and I is the laser intensity By scaling the pressure, velocity, length, and temperature by ρ_0, V_0, L_0, and T_0, Eq. (1)-(3) give three nondimensional parameters, i.e., a Reynolds number $Re = \rho_0 V_0 L_0/\eta$, V_0/C_s, and η/κ where C_s is the sound speed.

We are interested in developing a model for the early time start-up phase pressure perturbation spectra from lasers smoothed by ISI methods. In a two-dimensional plane perpendicular to the direction of laser propagation, with $I(x,y) = \mathbf{E}^*\cdot\mathbf{E}$, an expression for the spectrum of laser intensity using ISI smoothing can be written[16]:

$$I\left(k\right) = \left(\pi - 2sin^{-1}k - 2k\left(1-k^2\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}\frac{I_{avg}}{2N_z} \quad (4)$$

where $k = (k_x^2 + k_y^2)^{1/2}$, $N_z = k_{max}D_{spot}/2\pi$, $k_{max} = 2\pi/f\lambda_0$, with D_{spot} the laser spot size, f is the f number, λ_0 is the laser wavelength, and I_{avg} is the average laser intensity. We note that the ISI smoothed laser intensity deposit energy over a range of spatial scale sizes. As a result, from Eq. (3) to lowest order, the pressure perturbations will also be driven over a range of spatial scale sizes. Nonlinear effects may couple these spatial scales. Knowing the intensity spectrum I(x,y,z) the full spectral content of $\mathbf{J} \cdot \mathbf{E}$ from an ISI smoothed laser needs to be calculated in order to solve Eq. (3). We have computed the spectrum of $\mathbf{J} \cdot \mathbf{E}$ for ISI laser intensity of 10 - 50 TW/cm^2 and over a range of scales using a time-dependent Maxwell equation full wave code:

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{5}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} \tag{6}$$

$$\mathbf{J} = en\left(\mathbf{V}_i - \mathbf{V}_e\right) \tag{7}$$

where $\mathbf{V}_e$ and $\mathbf{V}_i$ are found from the electron and ion momentum equations. Fig. 2 shows the k-spectrum of $\mathbf{J} \cdot \mathbf{E}$ using the full wave Maxwell code. The laser was taken to be normally incident on the plasma slab configuration shown in Fig. 1.

We solve Eq. (1)-(3) numerically using a pseudospectral method. All transverse spatial derivatives are solved in Fourier k-space. The five independent variables are ρ, p, V_x, V_y, V_z. The time step algorithm is performed by a Runge-Kutta scheme. We use 128 x 128 x 64 grid points in the x,y,z directions, respectively. The boundary conditions are taken to be periodic in the transverse (x,y) directions and outflow in the z-direction. The simulation volume has dimensions of 60 μm, 60 μm, and 10

μm in the x,y,z directions. As initial conditions, our approach is to force Eq. (3) using spatial modes taken from the spectrum of $\mathbf{J} \cdot \mathbf{E}$ as computed near the critical surface using the full-wave Maxwell code of Eq. (5)-(7). We then follow the forced nonlinear hydrodynamic evolution and study the nonlinear coupling between the forced modes and the smaller scale modes. Aliasing errors are treated by truncating the Fourier values outside a shell of fixed wavenumber $k_{max} = 2^{1/2}N/3$ where N is the number of grid points in each direction. We take the following form for the forcing function:

$$S = \mathbf{J} \cdot \mathbf{E} = S_0 \left(A\left(t\right) sin \mathbf{k} \cdot \mathbf{x} + B\left(t\right) cos \mathbf{k} \cdot \mathbf{x} \right) \tag{8}$$

where A and B are random variables. Each element A and B is assumed to be statistically independent. The absolute magnitudes for S_0, A, and B are derived from the $\mathbf{J} \cdot \mathbf{E}$ spectrum computed from Eq. (5)-(7). In this study we force only the spatial modes $\mathbf{k}$ with $2\pi/50\mu m \leq |\mathbf{k}| \leq 2\pi/40\mu m$. This will allow the study of the cascading effects down to smaller scale modes to compare with experimental observations.

III. RESULTS

By defining the total energy by $E_T = E_I + E_K$ with $E_I = p/(\gamma - 1)$ and $E_K = (\rho/2)V^2$, we have found that the total and internal energy increase in time due to the ablation pressure applied by the $\mathbf{J} \cdot \mathbf{E}$ laser intensity profile but the kinetic energy stops increasing and oscillates about a quasiequilibrium value. We have also found that E_K is much smaller than E_I for the cases studied.

Fig. 3 shows the time development of the maximum and spatially averaged pressure. After an initial transient period the average pressure slowly increases linearly in time. Fig. 4 shows a plot of the pressure contours and illustrates the transverse (x,y) structure. Fig. 5 shows the time-averaged pressure spectrum. We have defined the pressure spectrum:

$$\frac{1}{2}\left(\frac{\tilde{p}}{p_0}\right)^2 = \int P\left(k\right)dk \tag{9}$$

with $\tilde{p} = p - p_0$. We have found that, for the cases studied, the presssure spectrum, from the simulation model, can be fit with a power law of the form $P(k) \simeq k^{-l}exp(-k^m)$ for the approximate scale size regime of 60 μm - 3 μm.

An approximate analytical expression for the pressure spectrum can be obtained as follows. In Eq. (3) we let $V_j = V_0 + \tilde{V}_j$, $S = S_0 + \tilde{S}_j$, and write:

$$\begin{aligned}\frac{\partial}{\partial t}\overline{\frac{1}{2}\tilde{p}^2} + V_j\frac{\partial}{\partial x_j}\overline{\frac{1}{2}\tilde{p}^2} = -\frac{\partial}{\partial x_j}\left(\overline{\frac{1}{2}\tilde{p}^2\tilde{V}_j} - \kappa\frac{\partial}{\partial x_j}\overline{\frac{1}{2}\tilde{p}^2}\right) \\ -\overline{\tilde{p}\tilde{V}_j}\frac{\partial p}{\partial x_j} - \kappa\overline{\frac{\partial\tilde{p}}{\partial x_j}\frac{\partial\tilde{p}}{\partial x_j}} + \overline{\tilde{p}\tilde{S}}\end{aligned} \tag{10}$$

where $\overline{f}$ denotes time average. We then take:

$$\frac{1}{2}\left(\frac{\tilde{V}}{V_0}\right)^2 = \int E\left(k\right)dk \tag{11}$$

We find that the pressure slowly increases in time reaching a quasiequilibrium. We take as an approximation $\partial/\partial t \simeq 0$ and balance the third and fourth terms on the right hand side and find:

$$P(k) \simeq k^{-p} exp\left(-Ak^q\right) \tag{12}$$

Here we have let $E(k) \simeq E_0 k^{-n}$ with p=2+(1-n)/2 and q=(n+1)/2. From the cases studied we find $n \simeq 1.5 - 2$, $p \simeq 1.5 - 1.8$, and $q \simeq 1.3 - 1.5$. We have found that the pressure spectrum is approximtely isotropic. This can be seen as follows. Eliminating the velocity from Eq. (2) and inserting into the pressure equation Eq. (3) we can write in $\mathbf{k}, \omega$ space, after ignoring source and sink terms, the lowest order nonlinear equation:

$$\frac{\partial \tilde{p}_{\mathbf{k},\omega}}{\partial t} = \sum_{\mathbf{k}'\omega'} T_{\mathbf{k},\mathbf{k}',\omega,\omega'} \tilde{p}_{\mathbf{k}',\omega'} \tilde{p}_{\mathbf{k}-\mathbf{k}',\omega-\omega'} \tag{13}$$

with

$$T_{\mathbf{k},\mathbf{k}',\omega,\omega'} = \frac{1}{\rho_0} \frac{i\left(\mathbf{k}\cdot\mathbf{k}' + (\gamma-1)\,k'^2\right)}{\omega' - i\eta k'^2} \tag{14}$$

and ρ_0 is an average density. We note that in the expression for the nonlinear matrix element T, which describes the nonlinear coupling to lowest order, for $\gamma > 1$ the second term is larger. The second term is function of $k' = |\mathbf{k}'|$ suggesting an approximately isotropic nonlinear coupling to lowest order.

We have found that the spectrum of pressure perturbations develops into a power law where large scales are dominant over smaller scales. Previous experimental studies in the start-up phase[6,8], using similar laser intensities and scale size regimes, have also found a power law dependence in k-space.

IV. SUMMARY

In summary, we have developed a nonlinear model for the spectrum of pressure perturbations due to optically smoothed lasers. This has been accomplished using a forced, dissipative model. The time dependent laser deposition has been computed for ISI using a full wave electromagnetic Maxwell code. We have found that the early time pressure spectra develops into a power law dependence where large scales are dominant over smaller scales. We have found that the model results are consistent with previous experimental observations. In the future we will study the pressure spectra from other spatial scale size forcings.

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Figure Captions

Fig. 1 Plot of initial density N and temperature T configuration.

Fig. 2 Plot of angle-averaged k-spectrum of $\mathbf{J} \cdot \mathbf{E}$. Normalizations of $S_0 = 10^{23}$ erg cm^{-3} sec^{-1} and $k_0 = 2\pi/60\mu m$.

Fig. 3 Plot of time history of pressure. The pressure has been normalized by $P_0 = 4$Mbar and t_0 =0.15 psec.

Fig. 4 Plot of normalized transverse pressure profile. The pressure here has been normalized by 5 Mbar. Eight evenly spaced contours have been plotted.

Fig. 5 Plot of pressure power spectra. Here $P_0 = 25$Mbar^2 and $k_0 = 2\pi/60\mu m$.

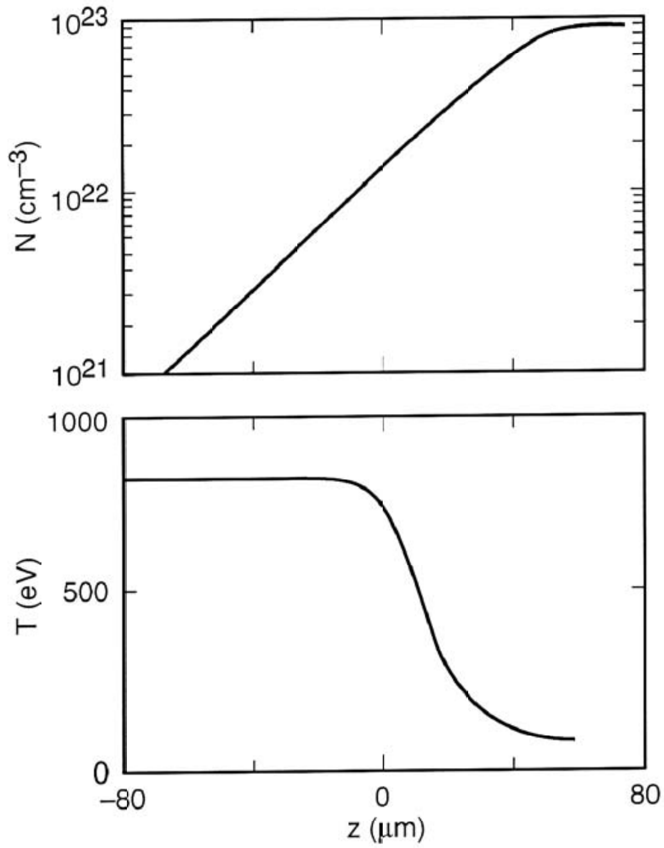

Fig. 1

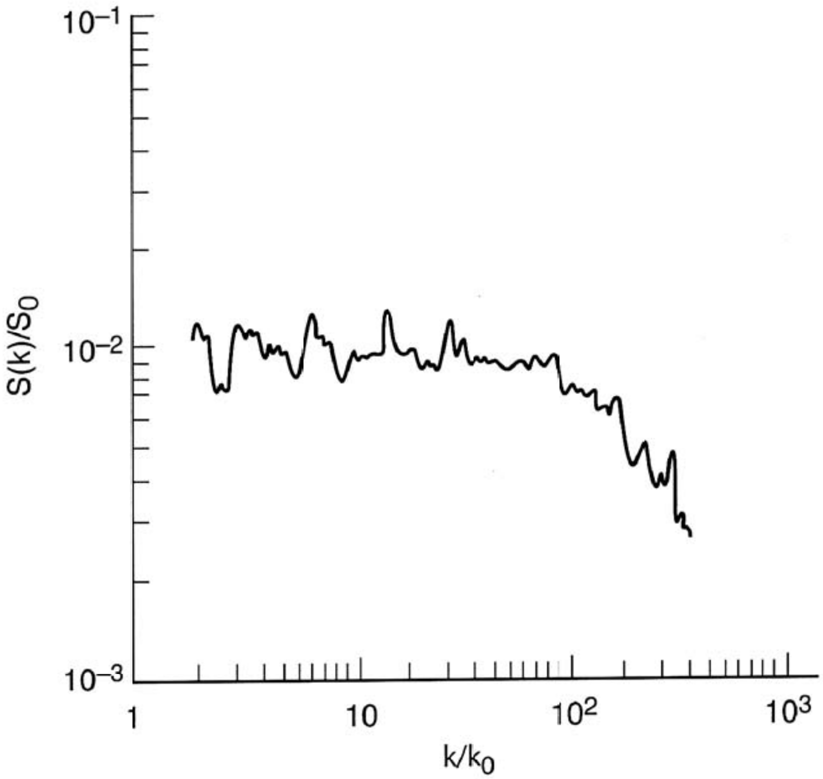

Fig. 2

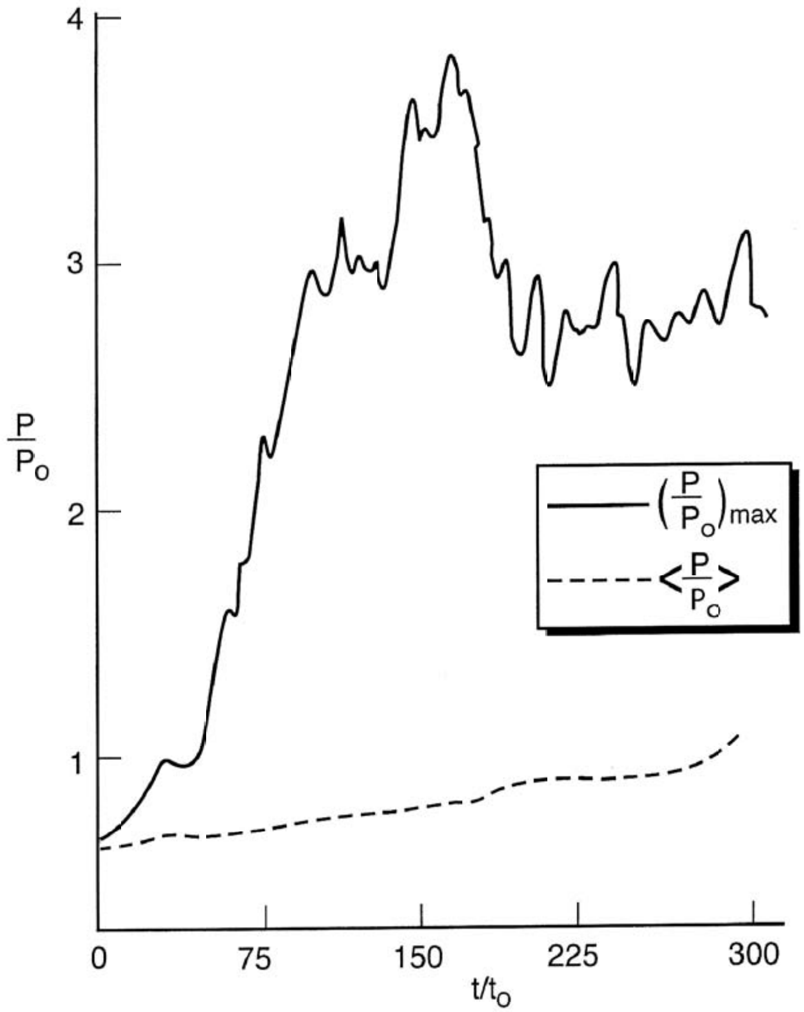
4
3
2
1
$\frac{P}{P_o}$
$\left(\frac{P}{P_o}\right)_{max}$
$\langle\frac{P}{P_o}\rangle$
0
75
150
225
300
t/t_o

Fig. 3

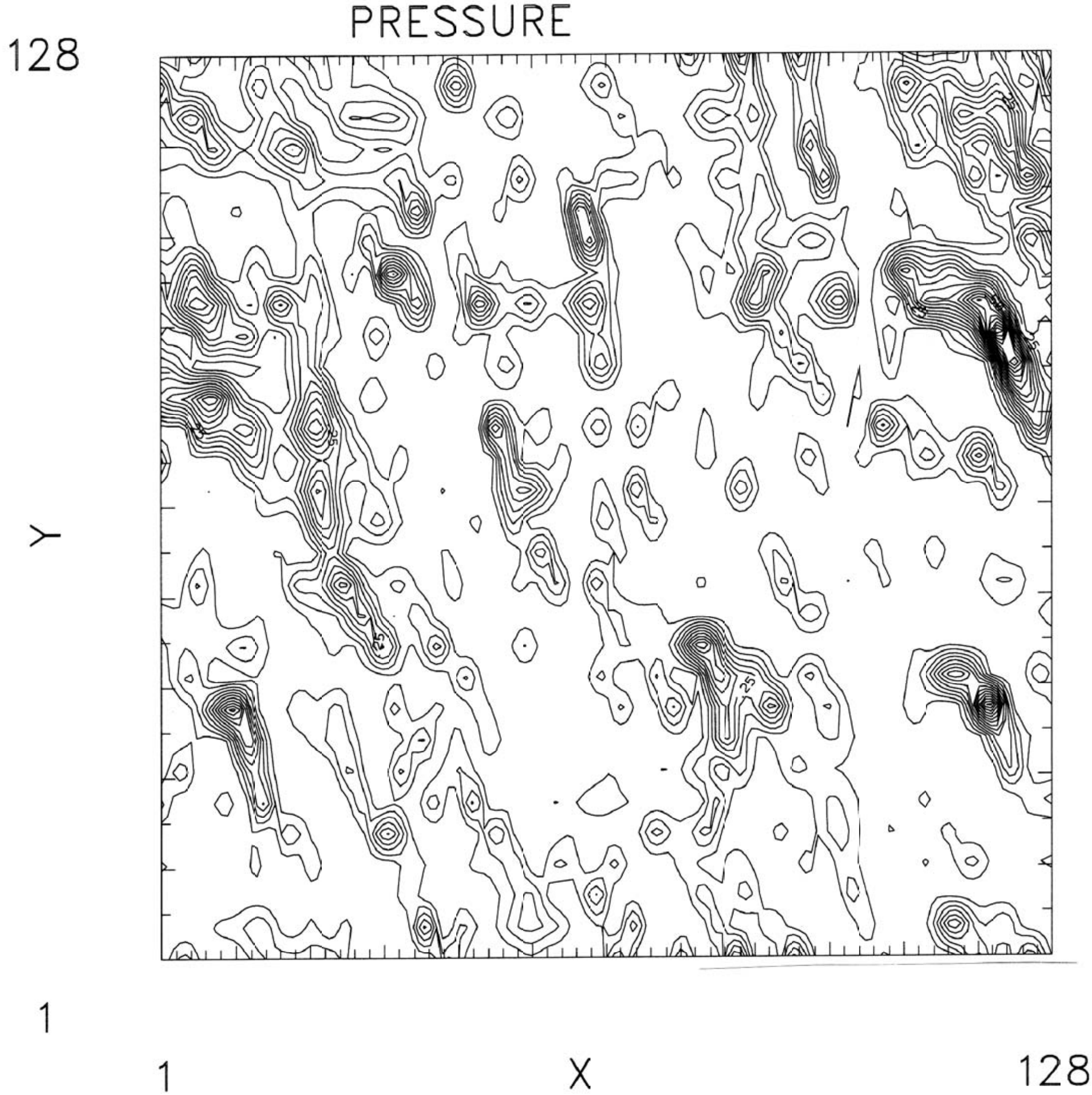

Fig. 4

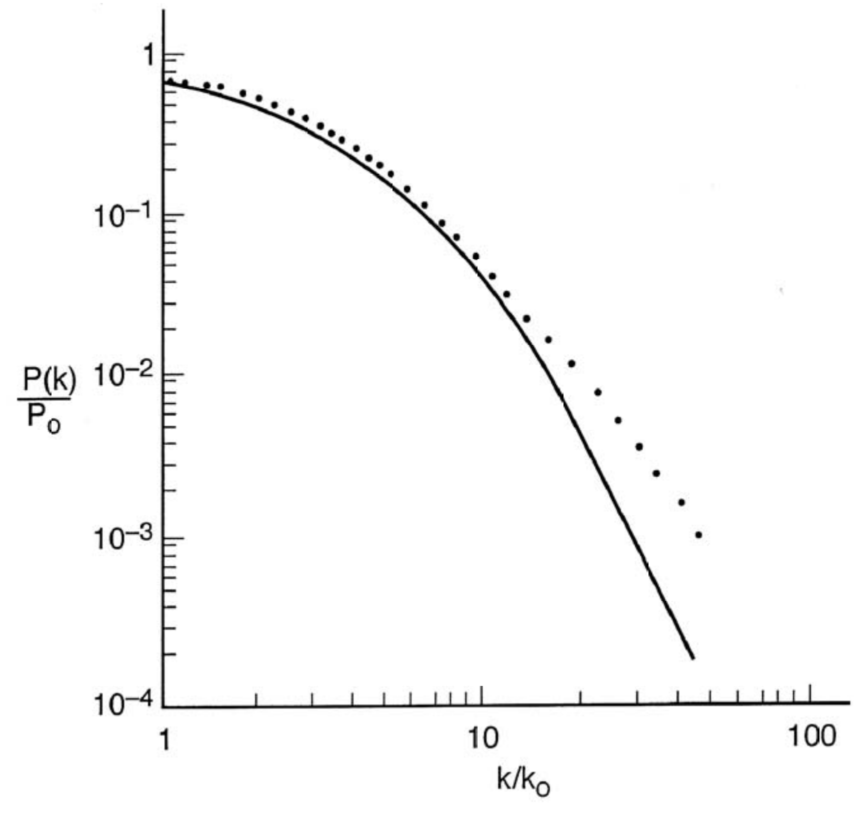

Fig. 5