Response to "Comment on 'Instability of isolated planar shock waves'" [Phys. Fluids 20, 029101 (2008)]

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In a recent article [J. W. Bates, "Instability of isolated planar shock waves," Phys. Fluids **19**, 094102 (2007)], we derived linear instability criteria for an isolated, planar, two-dimensional shock wave propagating through an inviscid fluid with an arbitrary equation of state. The basis for this analysis was a novel solution for the time-dependent Fourier amplitude of a single-mode perturbation on the front, which was expressed in the form of a Volterra equation. In the comment by Tumin ["Comment on 'Instability of isolated planar shock waves'," Phys. Fluids **20**, 029101 (2008)], the author demonstrated the consistency of our results with those of Erpenbeck, whose mathematical approach avoided the derivation of an integral equation in the time domain, but required a complicated, inverse Laplace-transform operation to ascertain the temporal evolution of disturbances at the shock's surface. Here, we emphasize that such information is obtained more readily from a direct solution of the aforementioned Volterra equation using modern numerical techniques.
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Recently,[1] we presented an alternate derivation of the well-known instability criteria for an isolated, planar, two-dimensional shock wave propagating through an inviscid fluid characterized by an arbitrary equation of state. The basis for this analysis was a novel solution in the linear approximation for the time-dependent Fourier amplitude of a single-mode perturbation on the front, which was expressed in the form of a Volterra equation of the second kind, and derived in a previous study.[2] In the comment by Tumin,[3] it was shown that our findings are consistent with those of Erpenbeck,[4] who also investigated the shock-wave instability problem, but whose differing mathematical technique sidestepped the formulation of a Volterra equation in the time domain. We would like to thank Tumin for his efforts in unifying these two disparate methodologies.

However, we wish to underscore an important aspect of the approach involving a Volterra equation. Using modern computational algorithms,[5,6] such an equation allows a simple and direct means of determining the evolution of disturbances at the shock's surface. For example, by numerically solving Eq. (9) of Ref. 1 for the normalized amplitude $g(\tau)$, one can demonstrate explicitly that linear perturbations on stable shock fronts oscillate, and then decay asymptotically as $\tau^{-3/2}$, where τ is a dimensionless time. This point is illustrated in Fig. 1, which shows the numerically computed solution of the shock-ripple amplitude for case "e" in Table I of Ref. 1, as well as the value of the decay envelope as a function of τ, plotted both linearly, and on a double-logarithmic scale. From the figure, it is clear that the data in logarithmic space approach a slope of $-3/2$ at late time. This implies that as $\tau \rightarrow \infty$, the temporal dependence of the function $g(\tau)/g(0)$ is approximately $\tau^{-3/2}$ (times a sinusoidal function of τ). Erpenbeck's method, by contrast, requires that a complicated, inverse Laplace-transform operation be performed to ascertain the same asymptotic behavior.

For shock conditions that satisfy the D'yakov-Kontovich criteria,[7,8] one can also show via a numerical solution of the Volterra equation in Eq. (9) of Ref. 1 that perturbations remain stationary. Two examples depicting this special class of shock instabilities were already presented in Fig. 4 of the aforementioned reference, and are labeled "c" and "d" in that plot. Upon inspection of those curves, it is clear that in each case the envelope of oscillations as a function of τ has zero slope asymptotically, which indicates that the perturbations do not evanesce over time (i.e., they are stationary). Furthermore, for absolutely unstable shock fronts — such as curves "a" and "b" in the same figure — the growth rates can be easily computed and shown to have the same values as the

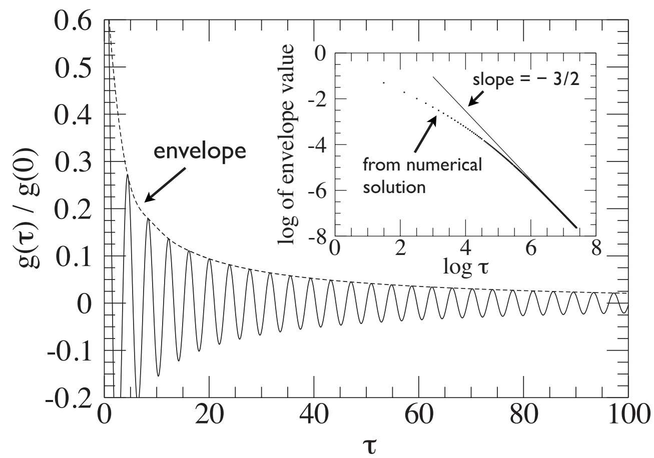

FIG. 1. Plot of the amplitude decay envelope of a linear sinusoidal perturbation on a stable shock front obtained by numerically solving the Volterra equation in Eq. (9) of Ref. 1. This figure also shows the value of the envelope as a function of time plotted on a double-logarithmic scale. The fact that the logarithmic data approach asymptotically a slope of $-3/2$ implies that $g(\tau)/g(0) \sim \tau^{-3/2}$ (times a sinusoidal function of τ) as $\tau \rightarrow \infty$.

positive real poles listed in Table I (6.216 and 0.059, respectively), in accordance with the linear theory.

In summary, the response of isolated planar shocks to linear perturbations can be investigated in one of several different ways, including a technique based on Riemann invariants due to Roberts,[1,2,9] a normal-mode analysis developed by D'yakov[7] and Kontorovich,[8] and another adopted by Erpenbeck.[4] Only the first and last methods, though, provide a theoretical framework for addressing the question of how stable shocks ultimately regain their planarity. Furthermore, although the consistency of these two approaches has been demonstrated recently by Tumin,[3] the existence of a time-dependent Volterra equation for the perturbation amplitude in the former methodology constitutes a significant advantage over the latter. By solving this equation numerically, the temporal evolution of small disturbances on isolated shocks propagating through fluids with arbitrary equations of state can be readily determined. In Erpenbeck's approach, the same information can be obtained only through a complicated, inverse Laplace-transform operation.

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[9] See National Technical Information Service Document No. PB2004-100597 (A. E. Roberts, "Stability of a steady plane shock," Los Alamos Scientific Laboratory Report No. LA-299, 1945). Copies may be ordered from National Technical Information Service, Springfield, VA 22161.